

# **Ribblesdale Federation of Schools**



## **Maths Calculations Policy**

**Reviewed by: (T Ward, January 2021)**  
**Approved by governors: February 2021**  
**Review date: January 2024**  
**Executive Headteacher: T Ward**  
**Chair of Governors: P. Gibbons**

Our Maths curriculum incorporates the use of high-quality mathematics work using a CPA approach (Concrete, Pictorial, Abstract), which are tailored to the needs of the learners in the school.

We provide opportunities to teach key mathematical strategies which support reasoning and problem solving. Mental and written calculation methods are taught alongside each other. When teaching children to calculate, emphasis is placed on choosing and using the approach that is the most efficient for the given situation and the child's ability to explain their working.

White Rose Maths Hub planning and resources are used as a starting point for our mathematics curriculum which is used to plan against age related expectations, the unique need of learners and the teaching of other subjects.

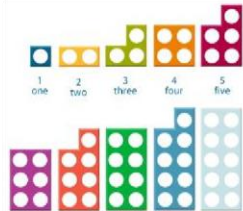
Children need to be able to explain, illustrate and justify relationships, patterns and generalisations within addition and subtraction using models and images to support their reasoning. Equipment and manipulatives should be used throughout all stages to support children in developing their ability to explain their thinking.

**Examples with Addition**

*Generalisations*

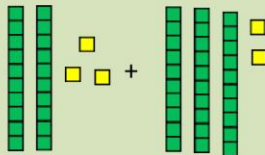
What are the rules for adding odd and even numbers? Experiment with different pairs of numbers to find a general rule for adding;  
 odd + odd  
 odd + even  
 even + even

Use these shapes to help you explain your reasoning



*Patterns and Reasoning*

Take any two digit number. Reverse the digits. Add the two numbers together. Write down your answer. Repeat with different two digit numbers. What do you notice about your answers? Does this always happen? Use Dienes to help you prove why.

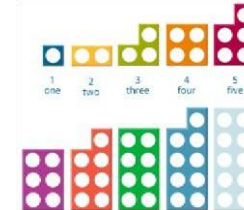


**Examples with Subtraction**

*Generalisations*

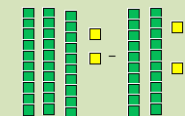
What are the rules for subtracting odd and even numbers? Are they the same as addition? Experiment with different pairs of numbers to find a general rule for adding;  
 odd - odd  
 odd - even  
 even - odd  
 even - even

Use these shapes to help you explain your reasoning


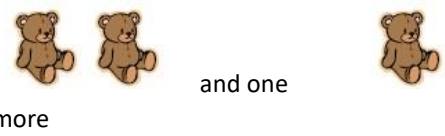
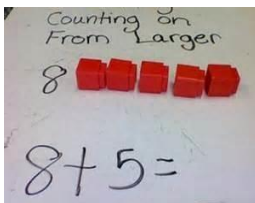

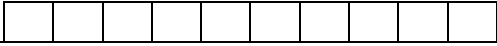
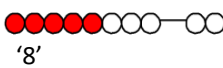



*Patterns and Reasoning*

Take any two digit number. Reverse the digits. Subtract the smaller number from the larger number. Write down your answer. Repeat with different two digit numbers. What do you notice about your answers? Does this always happen? Use Dienes to help you explain why.



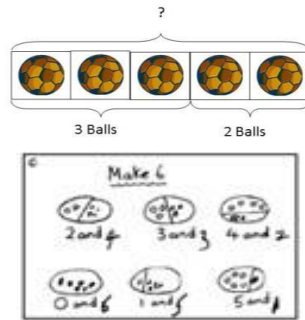
## ADDITION

Stage	Examples	Guidance and Notes										
<p><b>Stage 1: Developing and recording mental pictures</b></p> <p>Children should experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc.</p>	<p><u>Aggregation (combining two sets)</u></p> <p>Jen has three teddies. Jo has two teddies. How many teddies do they have altogether?</p>  <p><u>Augmentation (adding on to an existing set)</u></p> <p>How many teddies do we have?</p>  <p>How many will we have if we add one more?</p> <p>There are 3 people on the bus. Another 2 people get on. How many now?</p> <p><u>Counting on to support augmentation</u></p>  <p>Children should experience a range of progressively more abstract representations of number lines for counting on e.g. Number track</p> <table border="1" data-bbox="491 1339 1082 1400"> <tr> <td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td> </tr> </table> <p>Number line, all numbers labelled</p>  <p>Number lines, marked but unlabelled</p> 	1	2	3	4	5	6	7	8	9	10	<p>Initially recording of calculating should be done modelled by adults.</p> <p>Over time gradually introduce children to the recording process.</p> <p>Aggregation and augmentation should be taught alongside each other. Children should be encouraged to recognise the efficiency in 'counting on' as opposed to 'counting all'.</p> <p>Additional 'number lines' include the bead string which can be used to illustrate both aggregation and augmentation e.g.</p> <p><math>8 + 5 = 13</math></p>  <p>'8'</p>  <p>'8' + '5'</p>
1	2	3	4	5	6	7	8	9	10			

**Stage 2: developing additive number relationships**

This stage focuses on children developing a secure understanding of the relationships between numbers through a variety of models and approaches

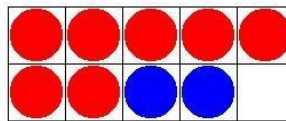
*Part, part, whole models*



*Ten frames*



Use frames to find 7+2



*Hundred square*

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Ensure children are exposed to opportunities to find number bonds for all whole numbers up to 20, including when confident, adding three numbers.

Ten frames support partitioning of all numbers up to 10, bridging through 10 and subitising numbers up to ten.

The hundred square can be used to support additive patterns in number as well as a tool for counting on (and back

– see subtraction stage 2).

**Stage 3: Develop understanding of using the empty number line**

The empty number line is intended to be a representation of a mental method, not a written algorithm. Therefore the order and size (physical and numerical) of the jumps should be expected to vary from one calculation to the next.

The empty number line helps to record the steps on the way to calculating the total.

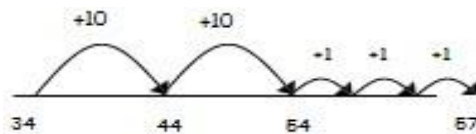
Steps in addition can be recorded on a number line

e.g.  $8 + 7 =$

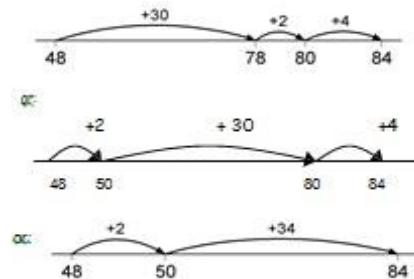


Extending to more efficient 'jumps' e.g. 34

$+ 23 =$



And e.g.  $48 + 36 =$



Children should be taught to use jumps of different sizes, and completed in an order that is most helpful depending on the numbers they are calculating with.

This reinforces that this is a visual representation of a mental method and not a written algorithm.

**Stage 4: Partitioning to support progression prior to introducing a formal written method**

Children need to be able to partition numbers in ways other than into tens and ones to support mental calculations.

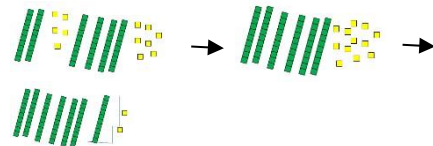
Partitioning into tens and ones will support progression to the columnar method for addition.

Children should use a range of practical apparatus (straws, Dienes apparatus, place value cards, place value counters) to support partitioning for addition progressing through gradually more abstract representations.

Straws, bundled into 10s and singularly allow children to see, create and count the '10' within the bundle.

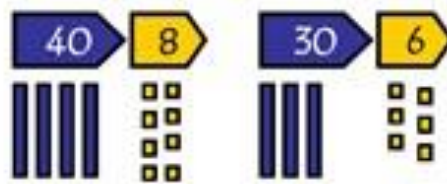
This then progresses to the use of Dienes (or similar) where 10s are clearly marked in ones but cannot be separated in the same way e.g.

$$25 + 47 =$$



Children need to have understanding of the size of number and the concept of one to many through multiplication before place value counters as these are a further abstraction as the '10' is labelled but not 'seen'.

$$48 + 36$$



$$40 + 30 = 70$$

$$8 + 6 = 14$$

$$70 + 14 = 84$$

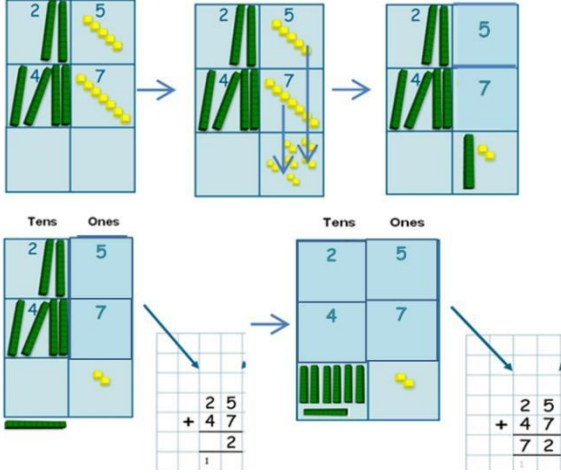
Money should also be used (1ps, 10ps and £1) as place value equipment to help children develop their understanding of a range of representations.

Children may make jottings to support their thinking during at the beginning of this stage.

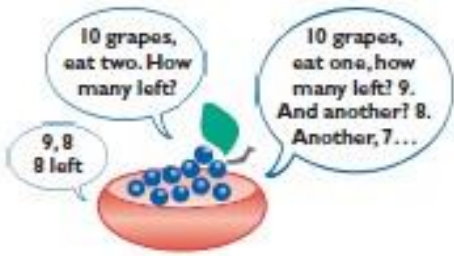
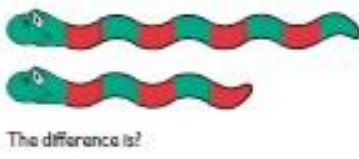
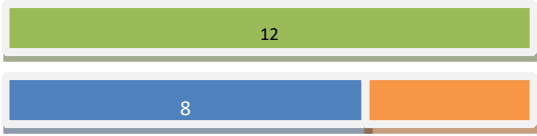
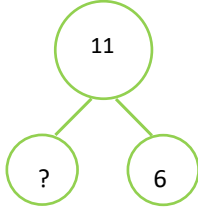
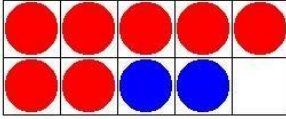
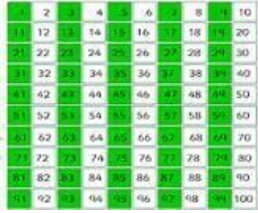
This stage explores the processes practically until they are secure with the concept before the written recording is introduced (stage 5).

Children should use the term 'exchange' or 'regroup' to describe converting ten ones into one ten rather than 'carrying'.

Choose calculations carefully to ensure the size of numbers do not distract focus from the concept.

<p><b>Stage 5: Using Dienes alongside columnar written method</b></p> <p>To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout.</p>	<p>It may be appropriate to teach children the process with numbers that they would be more appropriate to calculate mentally or with jottings to aid with the practicalities of the use of such equipment. However this should be the exception rather than the rule so children see a clear purpose for learning a new method for calculating e.g. <math>25 + 47 =</math></p>  <p>The diagram illustrates the process of adding 25 + 47 using Dienes blocks and a columnar grid. It shows the exchange of ten ones for one ten, resulting in 72.</p>	<p>Children should first experience the practical version of column addition and move to recording the written method alongside when they are confident in the process and once modelled by the teacher.</p> <p>Children should experience this stage with a variety of practical equipment to make sure their understanding is embedded and transferrable between representations.</p> <p>When the tens barriers is crossed 'exchange' or 'regrouping' then takes place.</p>								
<p><b>Stage 6: Securing the compact column method</b></p> <p>Extend to more complex combinations such as three two-digit numbers, two three-digit numbers, and problems involving several numbers of different sizes, including decimals.</p>	<table style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding-right: 20px;"><math>258</math></td> <td><math>366</math></td> </tr> <tr> <td style="padding-right: 20px;"><math>+ 87</math></td> <td><math>+ 458</math></td> </tr> <tr> <td style="padding-right: 20px;"><math>\hline 345</math></td> <td><math>\hline 824</math></td> </tr> <tr> <td style="padding-right: 20px;"><math>11</math></td> <td><math>11</math></td> </tr> </table> <p>Extend to four-digit, add four-digit and more complex combinations such as several numbers to be added, problems involving several numbers of different sizes and further to include decimals.</p>	$258$	$366$	$+ 87$	$+ 458$	$\hline 345$	$\hline 824$	$11$	$11$	<p>Schools should decide on a consistent approach to the location of exchanged digits and encourage children to be consistent with their placement to reduce errors. However, understanding is more important than positioning and this should be made clear to children.</p>
$258$	$366$									
$+ 87$	$+ 458$									
$\hline 345$	$\hline 824$									
$11$	$11$									

## SUBTRACTION

Stage	Examples	Guidance and Notes
<p><b>Stage 1: Recording and developing mental pictures</b></p> <p>Children are encouraged to develop a mental picture of the calculation in their heads. They experience practical activities using a variety of equipment and develop ways to record their findings including models and pictures.</p>	<p><i>Subtraction as take-away</i></p>  <p><i>Subtraction as finding the difference</i></p> 	<p>Initially recording of calculating should be done modelled by adults.</p> <p>Over time, gradually introduce children to the recording process.</p> <p>Children should be introduced to subtraction as take-away and difference, depending on the model that suits the story.</p>
<p><b>Stage 2: developing additive* number relationships</b></p> <p>This stage focuses on children developing a secure understanding of the relationships between numbers through a variety of models and approaches.</p> <p><i>*additive is the term which relates to both addition and subtraction</i></p>	<p><i>Part, part, whole models</i></p> <p><math>12 - 8 =</math></p>  <p><math>11 - 6 =</math></p>  <p><i>Ten frames</i></p> <p>Use the frame to help you calculate <math>9 - 2 =</math></p>  <p><i>Hundred square</i></p> 	<p>Ensure children are exposed to opportunities to use number bonds for all whole numbers up to 20 to help them find 'families of facts'</p> <p>Ten frames support number bonds to 10, bridging through 10 and subitising numbers up to ten (or 'distance' from 10).</p> <p>The hundred square can be used to support additive patterns in number as well as a tool for counting back.</p>



**Stage 3: Develop understanding of the number line**

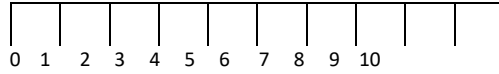
Children should develop a sound understanding of numbers to be able to use them confidently in calculation.

Mental calculation is supported by using number lines to support calculations and develop mental images.

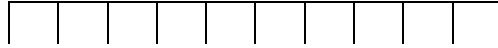
Children should experience a range of progressively more abstract representations of number lines e.g. Number track

1	2	3	4	5	6	7	8	9	10
---	---	---	---	---	---	---	---	---	----

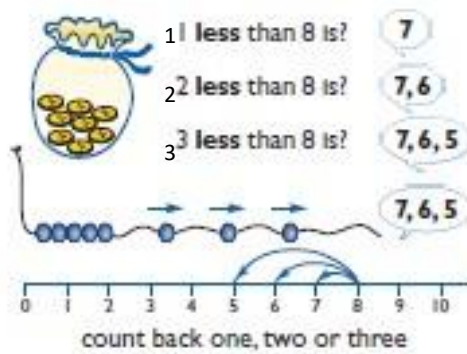
Number line, all numbers labelled



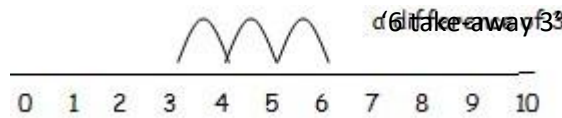
Number lines, marked but unlabelled



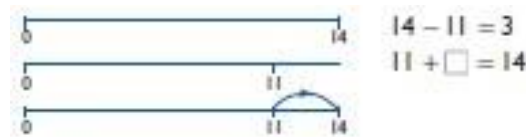
*Subtraction as take-away*



*Subtraction as take-away*



*Subtraction as the inverse of addition*



*Subtraction as finding the difference*



Additional 'number lines' - The bead string and hundred square.

A hundred square is an efficient visual resource to support subtraction.

Along with the number line, bead strings can also be used to illustrate subtraction e.g.

$$6 - 2 = 4$$



**Stage 4: Develop understanding of using the empty number line**

The empty number line is intended to be a representation of a mental method, not a written algorithm.

Therefore, the order and size (physical and numerical) of the jumps should be expected to vary from one calculation to the next.

The empty number line helps to record the steps on the way to calculating the total.

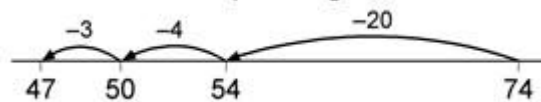
Counting Back

$$15 - 7 = 8$$

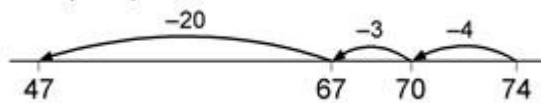
The seven is partitioned into 5 (to allow count back to 10) and two.



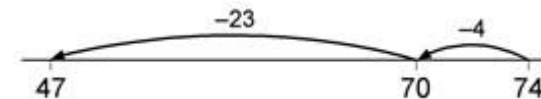
$74 - 27 = 47$  worked by counting back:



The steps may be recorded in a different order:

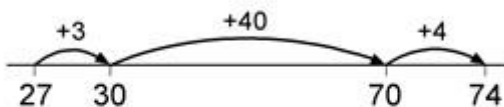


or combined



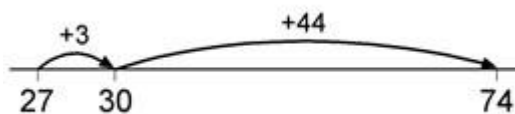
Counting on

$$74 - 27 =$$



The 'jumps' should be added, either mentally or with jottings according to confidence, beginning with the largest number e.g.  $40 + 4 + 3$ .

or



Progressing to choosing whether to count on or count back depending on what suits the numbers in the calculation e.g.



Children should be taught to use jumps of different sizes, and completed in an order that is most helpful depending on the numbers they are calculating with.

This reinforces that this is a visual representation of a mental method and not a written algorithm.

It is important that the empty number line is used for subtraction both as a mental model for supporting 'counting back' (to support the concepts of take-away and reduction) and for 'counting on' (to support the concepts of difference and subtraction as the inverse of addition)

**Stage 5: Partitioning to support progression to lead to a formal written method through takeaway**

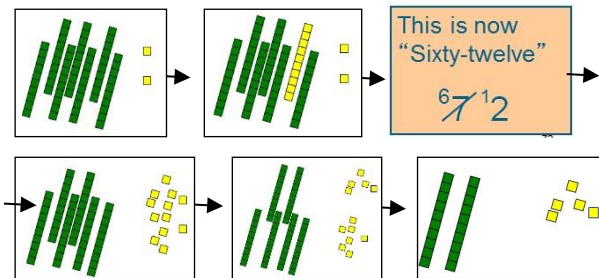
Children need to be able to partition numbers in ways other than into tens and ones to support mental calculations.

Partitioning into tens and ones will support progression to the columnar method for subtraction.

Children should use a range of practical apparatus (straws, Dienes apparatus, place value cards, place value counters) to support partitioning for subtraction progressing through gradually more abstract representations.

Straws, bundled into 10s and singularly allow children to see create and count the '10' within the bundle.

This then progresses to the use of Dienes (or similar) where 10s are clearly marked in ones but cannot be separated in the same way e.g.  $72 - 47 =$



Once children are able to use these with understanding, they will be able to progress to the use of place value cards and place value counters which are a further abstraction as the '10' is labelled but not 'seen'.

Money should also be used (1ps, 10ps and £1) as place value equipment to help children develop their understanding of a range of representations.

Children may make jottings to support their thinking during at the beginning of this stage.

This stage explores the processes practically until they are secure with the concept before the written recording is introduced (stage 5).

Children should use the term 'exchange' or 'regrouping' to describe converting ten ones into one ten rather than 'borrowing'.

Because of the cumbersome nature of 'exchanges' in this form, it may be helpful to limit examples that children are expected to do with the practical equipment limited to 3-digit take 3digit with one exchange in each calculation.

**Stage 6: Using Dienes alongside columnar written method**

To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout.

72 - 47

7 ones and then 4 tens are removed, leaving 25. The 25 can be dragged to the bottom to model the recording used in the written algorithm

Children should first experience the practical version of column subtraction and move to recording the written method alongside when they are confident in the process and once modelled by the teacher.

Children should experience this stage with a variety of practical equipment to make sure their understanding is embedded and transferrable between representations.

When the tens barrier is crossed 'exchange' or 're-group' takes place.

**Stage 7: Compact column method**

Extend to more complex combinations such as three two-digit numbers, two three-digit numbers, and problems involving several numbers of different sizes, including decimals.

$$\begin{array}{r} 51 \\ 563 \\ \hline 246 \\ \hline 317 \end{array}$$

932 - 457 becomes



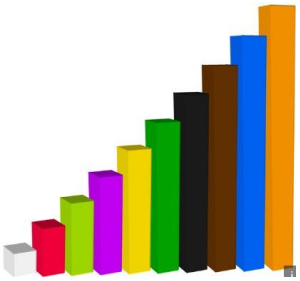
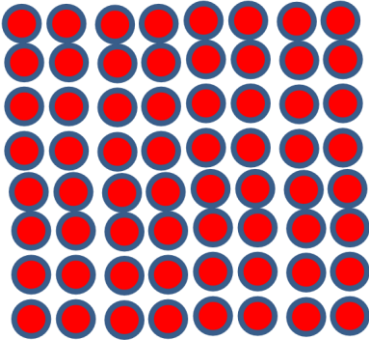
Children may find it more helpful to present their exchanges like this to keep the numbers clear.





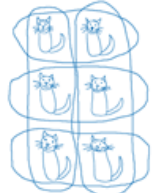


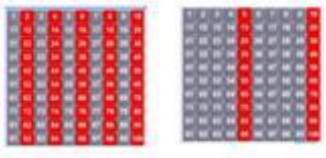
$$\begin{array}{r} 4 \quad 9 \quad 17 \\ 5 \quad 0 \quad 7 \\ \hline 1 \quad 8 \quad 9 \\ \hline 3 \quad 1 \quad 8 \end{array}$$

Children should continue to use practical equipment to support conceptual understanding until they are confident without it.

Schools should decide on a consistent approach to the location of exchanged digits and encourage children to be consistent with their placement to reduce errors. However, understanding is more important than positioning and this should be made clear to children.

Children need to be able explain, illustrate and justify relationships, patterns and generalisations within multiplication and division using models and images to support their reasoning. Equipment and manipulatives should be used throughout all stages to support children in developing their ability to explain their thinking.

Examples with Multiplication	Examples with Division
<p><i>Generalisations</i></p> <p>Sam says 'The total of three consecutive numbers is always a multiple of 3'. Is he correct?</p> <p>Use cubes to help you explain your reasoning</p> 	<p><i>Generalisations</i></p> <p>Use arrays to help you to explain why prime numbers only have one pair of factors.</p>  <p>Does this array represent a prime number?</p>
<p><i>Patterns and Reasoning</i></p>  <p>Select two numbers from 1 – 9 and find multiples which are common to both of them. What is the lowest common multiple? Try different pairs of numbers and find the lowest common multiple. What is the rule for calculating the lowest common multiple (LCM). Use Cuisenaire rods to explain your reasoning.</p>	<p><i>Patterns and Reasoning</i></p>  <p>Amelia thinks that if <math>64 \div 8 = 8</math> then <math>128 \div 16 = 16</math>.</p> <p>What do you think? Use place value counters to explain your reasoning</p>

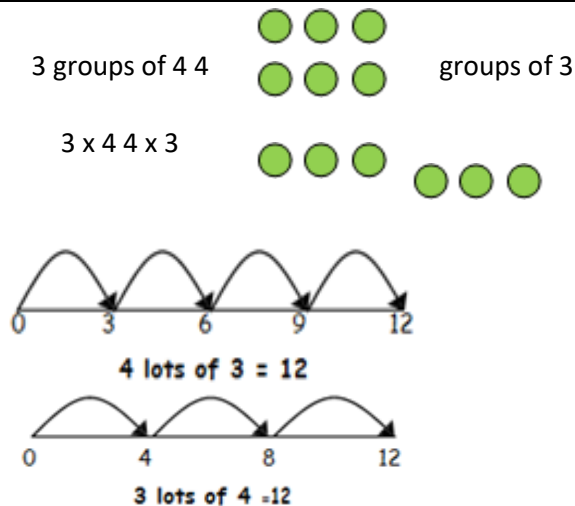
Stage	Examples	Guidance and Notes
<p><b>Stage 1: Recording and developing mental images</b></p> <p>Children should experience practical calculation opportunities involving equal sets or groups using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc.</p> <p>Children will count equal groups of objects. They will count in 2s and 10s and begin to count in 5s.</p> <p>Children will explore everyday versions of arrays such as egg boxes, baking trays, ice cube trays and wrapping paper</p> <p>Children will use repeated addition to carry out multiplication supported by the use of counters/cubes.</p>	<p style="text-align: center;"><math>2 + 2 + 2 + 2 + 2 = 10</math></p>   <p style="text-align: center;"><math>5 + 5 + 5 + 5 + 5 + 5 = 30</math> <math>5 \times 6 = 30</math></p> <p style="text-align: center;">6 groups of 5 are 30</p>  <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>(3 + 3) 2 groups of 3 are 6 (2 + 2 + 2) 3 groups of 2 are 6</p> </div>  <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>4 groups of 3 are 12 3 groups of 4 are 12</p> </div>  <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>Children should use pictorial representations and may use rings to show e.g. 3 groups of 2 and 2 groups of 3 introducing the commutative law of multiplication.</p> </div>	<p>Initially recording of calculating should be done by adults to model what children have done; using pictures, symbols, numbers and words.</p> <p>Over time there should be an expectation that children will also become involved in the recording process.</p> <p>Remember that the term 'lots of' may present a different image for children, using the words 'groups of' can be less confusing</p>
<p><b>Stage 2: Developing understanding multiplication as repeated addition, counting on the bead string, number line and hundred square</b></p> <p>Children continue to use repeated addition to carry out multiplication tasks and represent their counting on a bead string or a number line.</p>	<p>'Count out three groups of 5 then count the beads altogether'.</p> <p style="text-align: center;"><math>5 + 5 + 5 = 15</math></p>  <p style="text-align: center;"><math>10p + 10p + 10p + 10p + 10p = 30p</math> 5 hops of 10</p>  <p style="text-align: center;"><math>5 \times 10p = 50p</math></p> <p>Children explore patterns on a 100 square to help them begin to recognise multiples and rules of divisibility.</p>  <p style="text-align: center;">Multiples of 2      Multiples of 5</p>	<p>Songs and games provide opportunities to reinforce times tables facts and their associated patterns.</p> <p>These models illustrate how multiplication relates to repeated addition.</p>



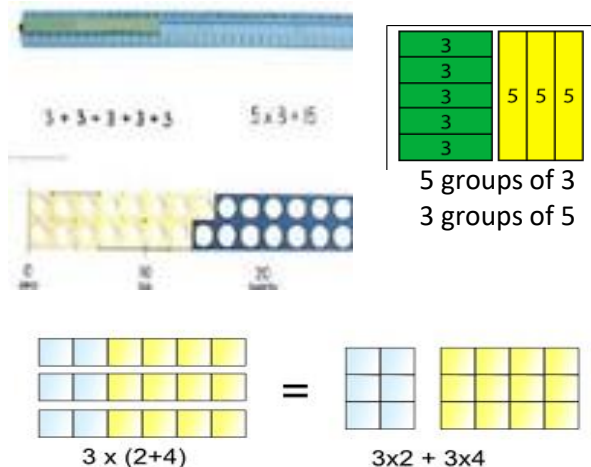
**Stage 3: Developing understanding of multiplicative relationships**

Children should be able to visualise multiplication as a rectangular array. This helps develop understanding of the commutative law

i.e.  $3 \times 4 = 4 \times 3$

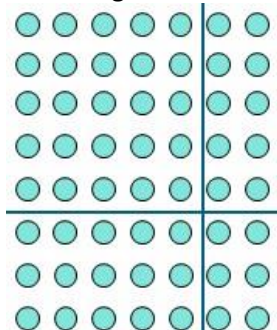


A rectangular array allows the total to be found by repeated addition and the link can be made to the 'x' sign and associated vocabulary 'groups of' or 'lots of'.



The outcome of  $3 \times 6$  will be the same as  $3 \times 6$  partitioned (in this example into 2 + 4)

Children use larger preconstructed arrays to look at ways these can be partitioned to use already know number facts e.g  $7 \times 8$



The relationship between the array and the number line showing both repeated additions should be demonstrated alongside each other

For more direct comparison, this could then be demonstrated on a single number line as appropriate.

Cuisenaire rods and Numicon can be used on a number line to develop understanding of multiplication as repeated addition and can be made into arrays.

Children should partition arrays in a variety of helpful ways which are not necessarily the ways in which they will eventually partition them to be in line with formal written methods

This is the first exposure to the distributive law of multiplication and children should be given plenty of opportunity to explore this.

**Stage 4: Using the Grid Method to multiply by a single digit number.**

$4 \times 13 =$



This then becomes

x	10	3
4	40	12

$40 + 12 = 52$

They begin to represent record in a column format alongside the grid method.

$$\begin{array}{r} 13 \\ \times 4 \\ \hline 52 \end{array}$$

Some children may need an expanded form of recording to support their understanding of the formal method.

$$\begin{array}{r} 13 \\ \times 4 \\ \hline 12 \quad 4 \times 3 \\ 40 \\ 52 \quad 4 \times 10 \end{array}$$

Recording is reduced further with the carried digits recorded either below the line or at the top of the next column. **324 x 7 becomes**

$\begin{array}{r} 21 \\ 342 \\ \times \quad 7 \\ \hline 2394 \end{array}$	$\begin{array}{r} 342 \\ 342 \\ \times \quad 7 \\ \hline 2394 \end{array}$
---	--

This example shows the carried digits below the line

This example shows the carried digits at the top of the next column

The link between arrays and the grid method should be made clear to children by the use of place value apparatus such as place value counters and Dienes.

Children should be able to identify and use related calculations and place value effectively

Children should be moved towards starting with the column of smallest value as soon as their understanding of the relationship between the methods allows, to move towards long multiplication.

To develop efficiency children should know multiplication facts up to  $10 \times 10$

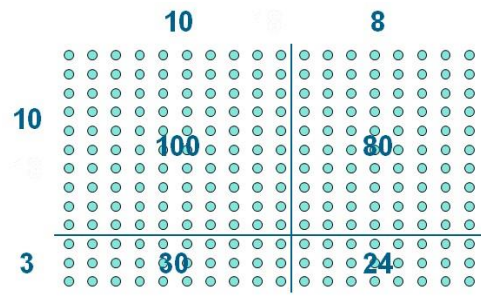
The expectations of the national curriculum are that children will know all tables up to  $12 \times 12$

'Carrying' can be done above or below the number, but should be consistent as before to avoid mistakes.



**Stage 5: multiplying by a two-digit using the grid method (TU x TU)**

18 x 13 =



Children move to the grid method without arrays once they can confidently explain the relationship between the two, even when the array is no longer visible.

		10	8
x			
10	100		80
3	30		24

They begin to represent the method of recording in a column format.

18

X 13

54

180

234

Encouraging children to discuss and compare the 3 x 18 10 x 18 two methods develops understanding

Each digit continues to be multiplied by each digit, but the totals are recorded in a more compact form, using 'carrying'.

124 x 26 becomes

```

    1 2
  1 2 4
×   2 6
-----
  7 4 4
 2 4 8 0
 3 2 2 4
-----
11
  
```

Answer: 3224

Before carrying out calculations children should be encouraged to estimate their answer using rounding.

They should compare their answer with the estimate to check for reasonableness.

**Adding the rows or adding the columns?**

This should be decided by the child depending on the numbers that are produced through the calculation.

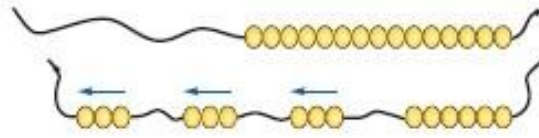
Schools should decide on a consistent approach to the location of 'carried' digits and encourage children to be consistent with their placement to reduce error rates. However as long as children understand the carried digit, the location of the recording is unimportant and this should be made clear to children.

**DIVISION**

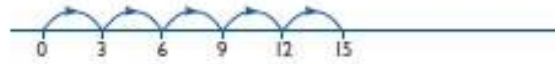
Stage	Examples	Guidance and Notes
<p><b>Stage 1: Recording and developing mental images</b></p> <p>Children are encouraged through practical experience to develop physical and mental images. They make recordings of their work as they solve problems where they want to make equal groups of items or sharing objects out equally.</p>	<div data-bbox="544 203 948 645" data-label="Image"> <p>12 apples to share with 3 friends equally</p> </div> <p><b>Sharing and grouping</b></p> <p>Children solve problems by <b>sharing</b></p> <p>15 eggs are shared between 5 baskets. How many in each basket?</p> <div data-bbox="539 936 986 1093" data-label="Diagram"> </div> <p><i>First egg to the first basket, 2<sup>nd</sup> egg to the second etc</i></p> <p>Children solve problems by creating <b>groups</b> of a given number.</p> <p>There are 15 eggs. How many baskets can we make with 3 eggs in?</p> <div data-bbox="507 1532 986 1756" data-label="Image"> </div>	<p>Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process.</p> <p>Children solve <b>sharing problems</b> by using a 'one for you, one for me' strategy until all of the items have been given out.</p> <p>Children should find the answer by counting how many eggs <b>1 basket</b> has got.</p> <p>Children should find the answer by counting out the eggs and finding out how many <b>groups of 3</b> there are.</p>

**Stage 2: children use equal step counting to support division**

15 eggs are placed in baskets, with 3 in each basket. How many baskets are needed?



Counting on a labelled and then blank number lines.  $15 \div 3 = 5$



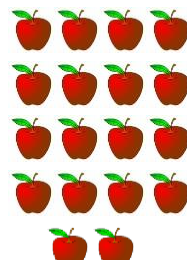
Using a bead string or number line children can represent division problems

Children count on in equal steps based on adding multiples up to the number to be divided.

Numicon and Cuisenaire rods can be used on a number line to develop understanding.

**Stage 3: Developing understanding of multiplicative relationships**

$3 \times 4 = 12$   
 $4 \times 3 = 12$   
 $12 \div 4 = 3$   
 $12 \div 3 = 4$



**Grouping:** If we put 4 apples in each bag how many bags will be full?

How many will be in the bag that is not full?

How many bags will we need?

**Sharing** many apples will ea  
ow can we share the two  
get? t between four children?

How m



ould each child  
ave?

$12 \div 2 = 6$

$\frac{1}{2}$  of 12 = 6

$12 \div 4 = 3$

$\frac{1}{4}$  of 12 = 3

Arrays support children in seeing the links between grouping and sharing and between multiplication and division.

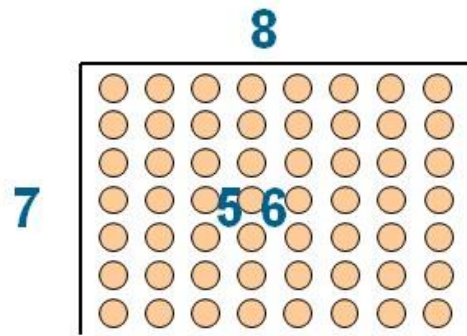
Children continue to solve problems by **grouping** or by **sharing** developing an understanding of remainders in each context. They understand when a remainder can be expressed as a fraction.

Children begin to make connections between division and fractions

**Stage 4: Using arrays to support children in moving towards standard written methods for division**

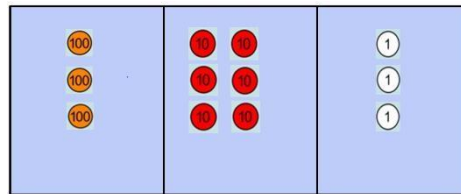
Children construct arrays by grouping the dividend into groups of the divisor. The number of groups made is recorded as the quotient.

Children then begin to construct the arrays using place value equipment to represent the dividend.

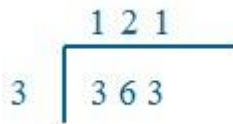


Divided  $(56) \div$  divisor  $(7) =$  Quotient  $(8)$

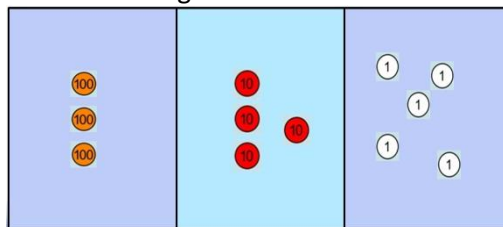
Using the principles of arrays linked to place value  $363 \div 3$  becomes:



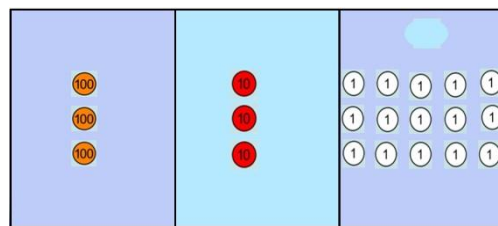
Each part of the number is grouped or shared into the divisor. Explaining the recording of the division as;



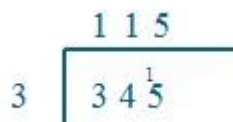
This then becomes more complex when exchange is needed as complete groups of the divisor cannot be made e.g.



Then becomes



Recorded as



The use of arrays help to reinforce the link between multiplication and division

This can then be explained in two ways;

In one of the three groups, there is one hundred, two tens and one one, making one hundred and twentyone

OR

There is 1 group of three hundreds, 2 groups of three tens and 1 group of three ones making one hundred and twenty-one

### Stage 5: Short and Long division

Once children have developed a sound understanding of division, using the manipulatives 'formal written methods' of short and then long division can be introduced.

#### Short division

With short division, children are expected to 'internalise' the working from **Stage 4**

$432 \div 5$  becomes

$$\begin{array}{r} 86r2 \\ 5 \overline{)432} \end{array}$$

Answer 86 remainder 2

#### Long Division

$432 \div 15$  becomes

$$\begin{array}{r} 28 \\ 15 \overline{)432} \\ \underline{300} \\ 132 \\ \underline{120} \\ 12 \end{array}$$

Answer 28 remainder 12

Children may choose to record the 'chunks' alongside to help them calculate the final answer

$432 \div 15$  becomes

$$\begin{array}{r} 28 \\ 15 \overline{)432} \\ \underline{300} \quad 15 \times 20 \\ 132 \\ \underline{120} \quad 15 \times 8 \\ 12 \end{array}$$

Answer 28 remainder 12

$$12 = 4$$

$$\begin{array}{r} \underline{\quad} \\ 15 \end{array} \quad \begin{array}{r} \underline{\quad} \\ 5 \end{array}$$

Children will start to interpret the 'remainder' in the most appropriate way to the context of the question.

For calculations where numbers with up to 4 digits are divided by a single digit number, children are expected to use **short division**.

For calculations where numbers of up to 4 digits are divided by a two-digit number, children are expected to use **long division**.

By the time children are ready for long division, manipulatives may not aid calculating, however they may aid the understanding of the process of long division.



## Progression of Mental Calculation

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Autumn 1	Count numbers to 20, forwards and backwards	Count in multiples of 2, 5, 10	Count in multiples of 3	Count in steps of 10, 100 and 1000 from any number	Count forwards and backwards in powers of 10 from any number	Revision from years 3 - 5
	Read and write numbers to 20 in numerals Number bonds to 5 Related subtraction facts	Number bonds from 20 to 100 (in multiples of 10)	Revise $\times$ and $\div$ 2, 5 and 10 Number bonds to 1000 (in multiples of 100 then 50)	Revise $\times$ and $\div$ 2, 4 and 8	Recall the prime numbers up to 19	Additive relationships for $90^0$ , $180^0$ and $360^0$ Complements to 1000, 100, 10 and 1
Autumn 2	Count numbers to 50, forwards and backwards	Count in multiples of 2, 5 and 3 Count in steps of 10 from any number	Count from 0 in multiples of 4 and 8	Count from 0 in multiples of 6, 7 and 9	Count backwards through 0 to negative numbers	Use double number lines to count in approximate conversions for metric and Imperial measures including miles to kilometres
	Read and write numbers to 50 in numerals Number bonds to 10 (emphasise subitising to 5 e.g. $8 = 5 + 3$ ) Related subtraction facts	Number bonds from 20 to 100 (in multiples of 5 and 10)	Add and subtract mentally: Three-digit numbers and one-digit; Three-digit numbers and 10s; Three-digit numbers and 100s	$\times$ and $\div$ 3, 6, 9 and 7	Square numbers to $15^2$ and multiples of 10 to $100^2$ Cube numbers to $5^3$ and also $10^3$	Using known $\times$ and $\div$ facts to support calculation  Use knowledge of rules of divisibility??
Spring 1	Count numbers to 100, forwards and backwards	Count in multiples of 3 Count in even numbers up to 50 and odd numbers up to 30, forwards and backwards	Count from 0 in multiples of 50 and 100	Count from 0 in multiples of 25	Count forwards and backwards in powers of 10 from any number and through 0 to negative numbers	Revision from years 3 - 5
	Read and write numbers to 100 Number bonds to 20 Related subtraction facts	Number bonds from 20 to 100 Add and subtract mentally: 3 one-digit numbers	$\times$ and $\div$ 3	$\times$ and $\div$ all to $12 \times 12$	Use place value to add and subtract large numbers mentally	
Spring 2	Count in multiples of 2	Count in steps of $\frac{1}{2}$ and $\frac{1}{4}$ up to 10 (including context of time)	Revise counting in steps of $\frac{1}{2}$ , $\frac{1}{4}$ and $\frac{1}{3}$ Count in 1/10s, forwards and backwards	Count in multiples of 60 from 0 (to relate to time conversions)	Count in unit fractions including 1/10s and 1/100s, bridging through zero.	Revision from years 3 - 5
	Facts families to 20 (e.g. $8+7=15$ , $7+8=15$ , $15-7=8$ , $15-8=7$ )	Add and subtract mentally: Two-digit numbers and one-digit; Two-digit numbers and 10s	$\times$ and $\div$ 2, 4 and 8	$\times$ and $\div$ all to $12 \times 12$	$\times$ and $\div$ whole numbers and decimals by powers of 10	
Summer 1	Count in multiples of 2 and 10	Count in steps of $\frac{1}{3}$ up to 10	Count in decimal tenths	Revise counting in 1/10s and other unit fractions Count in 1/100s	Count in decimals, bridging through zero	
	Doubles of numbers to 10 and corresponding halves	$\times$ and $\div$ 2 and 10 Add and subtract mentally: 2 two-digit numbers (initially without bridging followed by bridging)		Know and use factor pairs	$\times$ and $\div$ numbers mentally using known facts	
Summer 2	Count in multiples of 2, 10 and 5	Revisit aspects from Y1 and Y2	Count in coin values (including 20)	Count in decimals, forwards and backwards	Counting in units of time (e.g. 7 days, 30 minutes)	

					<i>Counting forwards and backwards in minutes across o'clock)</i>	
	<i>Doubles of numbers to 20 and corresponding halves</i>	<i>× and ÷ 2, 10 and 5</i>	<i>× and ÷ 5,10; 2, 4, 8</i>		<i>Bridge across 60 when calculating time</i>	